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RIKATTI TENGLAMASI YECHIMINING HOSILALARI UCHUN REKURRENT-DIFFERENSIAL TENGLAMALAR SISTEMASI

Abstract: This is $y'' + xy = 0$ equivalent to the Liouville $z'(x) = x + z^2(x)$ equation. The system of recurrent-differential equations for the derivatives of the solution of the Riccati equation is studied.

Key words: recurrent-differential equation, polynomial triangle, infinite system.

Annotatsiya: Ushbu $y'' + xy = 0$ Liuvill tenglamasiga $z'(x) = x + z^2(x)$ tengkuchli. Rikkati tenglamasi yechimining hosilalari uchun rekurrent-differensial tenglamalar sistemasi o'rganiladi.

Kalit so'zlar: rekurrent-differensial tenglama, polinomial uchburchak, cheksiz sistema.

Аннотация: Это эквивалентно $y'' + xy = 0$ уравнению Лиувилля. Исследована $z'(x) = x + z^2(x)$ система рекуррентно-дифференциальных уравнений для производных решения уравнения Риккати.

Ключевые слова: рекуррентно-дифференциальное уравнение, полиномиальный треугольник, бесконечная система.

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Parametrga bog‘liq chiziqli tenglamalarni kichik parametr usuli bilan

$$y'_n(x) = a_n(x)y_n(x) + b_n(x)y_{n-1}(x) + c_n(x), n = 0, 1, 2, \dots \quad (1)$$

ko‘rinishidagi rekurrent-differensial tenglamalar cheksiz sistemasiga keltirish mumkinligi yaxshi ma‘lum, bunda $y_0(x)$ berilgan deb qaraladi. Bu sistema aslida sof rekurrent tabiatli bo‘lib, no‘malumlar birin-ketin integrallash yo‘li hisoblanadi.

Amaliyotda (1) sistemadan farqli

$$P_{n+1,k}(x) = a_{n,k}(x)P_{n,k}(x) + b_{n,k}(x)(P_{n,k-1}(x))' + c_{n,k}(x)P_{n,k-2}(x) \quad (2)$$

ko‘rinishdagi tenglamalar sistemasiga keltiriladigan masalalar uchraydi. Xususan, $a_{n,k}(x) = n - k, b_{n,k}(x) = 1, c_{n,k}(x) = (n - k + 2)x$ bo‘lgan hol Liuvill tenglamasi uchun kombinatorik masalaga teng kuchli. Boshlang‘ich shart $P_{n,0} = n!$ bo‘lgan holni qaraymiz (bu yerda $k < n$). Shunda $P_{n,k}(x)$ ko‘phadlardan iborat bo‘lib, u binomial koeffitsientlar kabi uchburchak hosil qiladi.

			0				
		1	x				
	2	2x	1				
	6	8x	2	2x ²			
24	40x	10	16x ²	6x			
120	240x	60	136x ²	52x	16x ³ + 6		

(2) munosabatdan bevosita quyidagi xossalar kelib chiqadi: 1) Agar $k -$ juft bo‘lsa $P_{n,k}(x)$ bu $(\frac{k}{2} - 1)$ -darajali, agar toq bo‘lsa $([\frac{k}{2}] + 1)$ -darajali ko‘phad; 2) $d_n^k = \deg P_n^k(x)$ deyilsa, $d_{n+1}^{k+2} = d_n^k + 1, n = 2, 3, \dots, n - 1; k = 1, 2, \dots, n - 1$.

Endi $y'' + xy = 0$ tenglamani qaraymiz. Uning yechimi elementar funksiyalar orqali, ular ustida algebraik amallar va integrallash amali bilan ifodalanmaydi (J.Liuvil teoremasi) [1]. U

$$z'(x) = x + z^2(x) \quad (3)$$

Rikkati tenglamasiga teng kuchli.

Teorema. (3) tenglama uchun $z(0) = z_0$ Koshi masalasining yechimi $z(x)$

$$z^{(n)}(x) = n! z^{n+1} + \frac{1}{3}(n+1)! x z^{n-1} + \frac{1}{12}(n+1)! z^{n-2} +$$

$$+ \sum_{k=3}^n [(n-k+1)xP_{n-1,k-2} + (P_{n-1,k-1})' + (n-1 - k)P_{n-1,k}]z^{n-k}$$

xossaga ega.

Isbot. (3) tenglamadan $z'' = 2z^3 + 2xz + 1, z''' = 6z^4 + 8qz^2 + 2z + 2x^2$ bo'lishi ravshan[2].

So'ng

$$z^{(n)} = n!z^{n+1} + P_{n,1}z^{n-1} + P_{n,2}z^{n-2} + P_{n,3}z^{n-3} + \dots + P_{n,n-2}z^2 + P_{n,n-1}z^1 + P_{n,n} \tag{4}$$

o'rinli bo'lsin deb faraz qilaylik. U holda

$$\begin{aligned} z^{(n+1)} &= (n+1)!z^{n+2} + [(n-1)P_{n,1} + (n+1)!x]z^n + [(n-2)P_{n,2} + (P_{n,2})']z^{n-1} + \\ &+ [(n-1)xP_{n,1} + (n-3)P_{n,3} + (P_{n,2})']z^{n-2} + [(n-2)xP_{n,2} + (n-4)P_{n,4} + (P_{n,3})']z^{n-3} + (5) \\ &+ [(n-3)xP_{n,3} + (n-5)P_{n,5} + (P_{n,4})']z^{n-4} + [(n-4)xP_{n,4} + (n-6)P_{n,6} + (P_{n,5})']z^{n-5} + \\ &+ [(n-k+2)xP_{n,k-2} + (n-k)P_{n,k} + (P_{n,k-1})']z^{n-k+1} + [(n-k+1)xP_{n,k-1} + (n-k-1)P_{n,k+1} + (P_{n,k})']z^{n-k} + \dots + [2xP_{n,n-2} + (P_{n,n-1})']z + xP_{n,n-1} + (P_{n,n})'. \end{aligned}$$

Bu ifodani (4) formulada n o'rniga $n+1$ qo'yilgani bilan taqqoslab, quyidagi tengliklarga ega bo'lamiz: $P_{n+1,0} = (n+1)!, P_{n+1,1} = (n+1)!x + (n-1)P_{n,1}, P_{n+1,2} = (n-$

$$2)P_{n,2} + (P_{n,1})', \text{ va } k = 3, \dots, n, P_{n+1,k} = (n-k+2)xP_{n,k-2} + (P_{n,k-1})' + (n-k)P_{n,k}.$$

Xususan, $P_{n+1,n+1} = xP_{n,n-1} + (P_{n,n})'$.

Boshlang'ich shart $P_{n,0} = n!$ bo'lgan holdan keyingi $P_{n,1}$ ko'phadni topamiz.

Yuqorida keltirilgan

$$P_{n,1} = n!x + (n-2)P_{n-1,1} \tag{6}$$

tenglamaning bir jinsli qismini quyidagicha yechamiz: $P_{n,1} = (n-2)P_{n-1,1}$ tenglamada $P_{n,1} = y_n$ kabi belgilash kiritsak, $y_n = (n-2)y_{n-1}$ ga ega bo'lamiz. Uning yechimi $y_n = (n-2)!$ cko'rinishda bo'ladi. O'zgarmasni variatsiyalash yo'li orqali

$$y_n = (n-2)!C_n \tag{7}$$

ifodani (6) tenglamaga qo'yamiz va C_n ga nisbatan $C_n = C_{n-1} + (n-1)nx$ rekurrent formulaga ega bo'lamiz. Keyingi qadamda n ga $1, 2, \dots, n$ qiymatlarni berib tenglamalar sistemasini hosil qilamiz va uni yechamiz: $C_n = (1 \cdot 2 + 2 \cdot 3 + \dots + (n-1) \cdot n)x = \frac{1}{3}n(n-1)(n+1)x$. C_n uchun topilgan ifodani (7) ga qo'yamiz va quyidagiga ega bo'lamiz :

$$P_{n,1} = y_n = (n-2)!C_n = \frac{1}{3}(n-2)!n(n-1)(n+1)x = \frac{1}{3}(n+1)!x.$$

Yuqoridagi qadamlarni davom ettirib,

$$P_{n,2} = \frac{1}{12}(n+1)!, P_{n,3} = \frac{1}{90}(n+1)!(5n-8)x^2,$$

$$P_{n,4} = \frac{1}{180}(n+1)!(5n-12)x, P_{n,5} = \frac{1}{5670}(n+1)!(35n^2 - 203n + 264)x^3 + \frac{1}{10080}(n+1)!(35n-106), \dots$$

larni hosil qilamiz.

Ushbu $P_{n,k} = (n-k+1)xP_{n-1,k-2} + (P_{n-1,k-1})' + (n-k-1)P_{n-1,k}$ lar $n > k \geq 3$ uchun o'rinli. Topilgan ifodalarni (4) ga qo'yib quyidagiga ega bo'lamiz:

$$\begin{aligned} z^{(n)} &= n!z^{n+1} + \frac{1}{3}(n+1)!xz^{n-1} + \frac{1}{12}(n+1)!z^{n-2} \\ &\quad + P_{n,3}z^{n-3} + \dots + P_{n,n-2}z^2 \\ &\quad + P_{n,n-1}z^1 + P_{n,n} = \\ &= n!z^{n+1} + \frac{1}{3}(n+1)!xz^{n-1} + \frac{1}{12}(n+1)!z^{n-2} \\ &\quad + \sum_{k=3}^n P_{n,k}z^{n-k} = n!z^{n+1} + \frac{1}{3}(n+1)!xz^{n-1} + \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{12}(n+1)!z^{n-2} \\ &\quad + \sum_{k=3}^n [(n-k+1)xP_{n-1,k-2} \\ &\quad + (P_{n-1,k-1})' + (n-1-k)P_{n-1,k}]z^{n-k}. \end{aligned}$$

Xulosa.

Ushbu maqolada Rikkati tenglamasi yechimining hosilalari uchun rekurrent-differensial tenglamalar sistemasi o'rganildi. Bunda $z'(x) = x + z^2(x)$ ko'rinishidagi tenglama uchun $z(0) = z_0$ Koshi masalasining yechimi $z(x)$

$$\begin{aligned} z^{(n)}(x) &= n!z^{n+1} + \frac{1}{3}(n+1)!xz^{n-1} + \frac{1}{12}(n+1)!z^{n-2} + \\ &\quad + \sum_{k=3}^n [(n-k+1)xP_{n-1,k-2} + (P_{n-1,k-1})' + (n-1-k)P_{n-1,k}]z^{n-k} \end{aligned}$$

xossaga ekanligi isbotlandi.

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